03 PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

BASIC CONCEPTS & FORMULAE

1. **Algebraic expression:** A combination of constants and variables, connected by four fundamental arithmetical operations $+, -, \times$ and $\div$ is called algebraic expression.

   For example, $3x^3 + 4xy - 5y^2$ is an algebraic expression.

2. **Equation:** An algebraic expression with equal to sign $(=)$ is called the equation. Without an equal to sign, it is an expression only.

   For example, $3x + 9 = 0$ is an equation, but only $3x + 9$ is an expression.

3. **Linear equation:** If the greatest exponent of the variable(s) in an equation is one, the equation is said to be a linear equation(s).

4. If the number of variables used in linear equation is one, then equation is said to be linear equation in one variable.

   For example, $3x + 4 = 0$, $3y + 15 = 0$; $2t + 15 = 0$; and so on.

5. If the number of variables used in linear equation is two, equation is said to be linear equation in two variables.

   For example, $3x + 2y = 12$; $4x + 6z = 24$, $3y + 4t = 15$, etc.

   Thus, equations of the form $ax + by + c = 0$, where a, b, c are non-zero real numbers (*i.e.*, $a, b \neq 0$) are called linear equations in two variables.

6. **Solution:** Solution(s) is/are the value/values for the variable(s) used in equation which make(s) the two sides of the equation equal.

7. Two linear equations of the form $ax + by + c = 0$, taken together form a system of linear equations, and pair of values of x and y satisfying each one of the given equation is called a solution of the system.

8. To get the solution of simultaneous linear equations two methods are used:

   (*i*) Graphical method (*ii*) Algebraic method

9. **Graphical Method**

   (*a*) If two or more pair of values for x and y which satisfy the given equation are joined on paper, we get the graph of the given equation.

   (*b*) Every solution $x = a$, $y = b$ (where a and b are real numbers), of the given equation determines a point (a, b) which lies on the graph of line.

   (*c*) Every point (c, d) lying on the line determines a solution $x = c$, $y = d$ of the given equation.

   Thus, line is known as the graph of the given equation.

   (*d*) When $a \neq 0$, $b = 0$ and $c \neq 0$ then the equation $ax + by + c = 0$ becomes $ax + c = 0$ or $x = -\frac{c}{a}$ then the graph of this equation is a **straight line parallel to y-axis** and passing through a point $\left(-\frac{c}{a}, 0\right)$.

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(*e*) When $a = 0$, $b \neq 0$ and $c \neq 0$ then the equation $ax + by + c = 0$ becomes $by + c = 0$ or $y = -\frac{c}{a}$ then the graph of the equation is a **straight line parallel to x-axis** and passing through the point $\left(0, -\frac{c}{b}\right)$.

(*f*) When $a \neq 0$, $b = 0$ and $c = 0$ then the equation is $ax = 0$ or $x = 0$. **Then the graph is y-axis itself.**

(*g*) When $a = 0$, $b \neq 0$, and $c = 0$ then equation becomes $by = 0$ or $y = 0$. Then the graph of this **equation is x-axis it self.**

(*h*) When only $c = 0$ then the equation becomes $ax + by = 0$. Then the graph of this **equation is a line passing through the origin.**

(*i*) The graph of x = **constant** is a line parallel to the y-axis.

(*j*) The graph of y = **constant** is a line parallel to the x-axis.

(*k*) The graph of $y = \pm x$ is a line passing through the origin.

(*l*) The graph of a pair of linear equations in two variables is represented by two lines.

(*i*) If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is **consistent.**

(*ii*) If the lines coincide, then there are infinitely many solutions—each point on the line being a solution. In this case, the pair of equations is also **consistent.**

(*iii*) If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is **inconsistent.**

10. Algebraic Method

(*a*) Substitution Method

(*b*) Method of Elimination

11. Conditions for solvability (or consistency)

If a pair of linear equations is given by $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then the following situations can arise :

(*i*) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

In this case, the pair of linear equations has a unique solution (consistent pair of equations)

(*ii*) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

In this case, the pair of linear equations has no solution (inconsistent pair of equations)

(*iii*) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

In this case, the pair of linear equations has infinitely many solutions [consistent pair of equations].

MULTIPLE CHOICE QUESTIONS

Choose and write the correct option in the following questions.

1. The value of k for which the lines represented by the following pair of linear equations are coincident is

$$2x + 3y + 7 = 0$$
$$8x + 12y + k = 0$$

(a) all real values except 14 (b) 8
(c) 28 (d) 14

2. The value of a for which the pair of equations $10x + 5y = a - 5$, $20x + 10y - a = 0$ has infinitely many solutions is

(a) 5 (b) –10 (c) 10 (d) 20

3. If the lines given by $3x + 2ky = 8$ and $2x + 5y - 4 = 0$ are parallel, then the value of k is

(a) $\frac{-5}{4}$ (b) $\frac{15}{4}$ (c) $\frac{2}{5}$ (d) $\frac{3}{2}$

4. The value of k for which the pair of equation $kx - y = 2$ and $6x - 2y = 3$ has unique solution

(a) $k = 3$ (b) $k \neq 3$ (c) $k \neq 0$ (d) $k = 0$

5. If a pair of linear equations has infinitely many solutions, then the lines representing them will be

(a) parallel (b) intersecting or coincident
(c) always intersecting (d) always coincident

6. The pair of linear equations $\frac{3x}{2} + \frac{5y}{3} = 7$ and $9x + 10y = 14$ is [*CBSE* 2020 (30/5/1)]

(a) consistent (b) inconsistent
(c) consistent with one solution (d) consistent with many solutions

7. The pair of linear equations $x = 0$ and $x = -4$ has [*CBSE* 2020 (30/4/1)]

(a) a unique solution (b) no solution
(c) infinitely many solutions (d) only solution (0, 0)

8. The pair of linear equations $y = 0$ and $y = -6$ has [*CBSE* 2020 (30/4/1)]

(a) a unique solution (b) no solution
(c) infinitely many solutions (d) only solution (0, 0)

9. The value of k, for which the pair of linear equations $kx + y = k^2$ and $x + ky = 1$ have infinitely many solutions is [*CBSE* 2020 (30/3/1)]

(a) ±1 (b) 1 (c) –1 (d) 2

10. The value of k for which the system of linear equations $x + 2y = 3$, $5x + ky + 7 = 0$ is inconsistent is [*CBSE* 2020 (30/2/1)]

(a) $-\frac{14}{3}$ (b) $\frac{2}{5}$ (c) 5 (d) 10

11. The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is [*CBSE* 2020 (30/1/1)]

(a) – 2 (b) $\neq 2$ (c) 3 (d) 2

12. For which value(s) of p, will the lines represented by the following pair of linear equations be parallel?

$$3x - y - 5 = 0$$
$$6x - 2y - p = 0$$

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(a) all real values except 10 (b) 10
(c) $\frac{5}{2}$ (d) $\frac{1}{2}$

13. The value of k for which the lines $(k + 1)x + 3ky + 15 = 0$ and $5x + ky + 5 = 0$ are coincident is

(a) 14 (b) 2 (c) –14 (d) –2